

Berechne A^{-1}

a) $A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ -1 & -1 & 1 \end{pmatrix}$

$$\begin{array}{ccc|ccc} 1 & 2 & -1 & | & 1 & 0 & 0 \\ 2 & 1 & 2 & | & 0 & 1 & 0 \\ -1 & -1 & 1 & | & 0 & 0 & 1 \end{array} \begin{array}{l} \xrightarrow{\cdot (-2)} \\ \xrightarrow{\cdot (-2)} \end{array}$$

$$\begin{array}{ccc|ccc} 1 & 2 & -1 & | & 1 & 0 & 0 \\ 0 & -3 & 4 & | & -2 & 1 & 0 \\ 0 & 1 & 0 & | & 1 & 0 & 1 \end{array} \begin{array}{l} \xrightarrow{\cdot (-2)} \\ \xrightarrow{\cdot 3} \end{array}$$

$$\begin{array}{ccc|ccc} 1 & 0 & -1 & | & -1 & 0 & -2 \\ 0 & 0 & 4 & | & 1 & 1 & 3 \\ 0 & 1 & 0 & | & 1 & 0 & 1 \end{array} \begin{array}{l} :4 \\ \updownarrow \end{array}$$

$$\begin{array}{ccc|ccc} 1 & 0 & -1 & | & -1 & 0 & -2 \\ 0 & 1 & 0 & | & 1 & 0 & 1 \\ 0 & 0 & 1 & | & \frac{1}{4} & \frac{1}{4} & \frac{3}{4} \end{array} \xrightarrow{\cdot (-1)}$$

$$\begin{array}{ccc|ccc} 1 & 0 & 0 & | & -\frac{3}{4} & \frac{1}{4} & -\frac{5}{4} \\ 0 & 1 & 0 & | & 1 & 0 & 1 \\ 0 & 0 & 1 & | & \frac{1}{4} & \frac{1}{4} & \frac{3}{4} \end{array}$$

$$A^{-1} = \begin{pmatrix} -\frac{3}{4} & \frac{1}{4} & -\frac{5}{4} \\ 1 & 0 & 1 \\ \frac{1}{4} & \frac{1}{4} & \frac{3}{4} \end{pmatrix}$$

b) $A = \begin{pmatrix} 1 & -1 & -2 \\ 2 & -1 & -3 \\ 3 & 2 & 1 \end{pmatrix}$

$$\begin{array}{ccc|ccc} 1 & -1 & -2 & | & 1 & 0 & 0 \\ 2 & -1 & -3 & | & 0 & 1 & 0 \\ 3 & 2 & 1 & | & 0 & 0 & 1 \end{array} \begin{array}{l} \xrightarrow{\cdot (-2)} \\ \xrightarrow{\cdot (-3)} \end{array}$$

$$\begin{array}{ccc|ccc}
 1 & -1 & -2 & | & 1 & 0 & 0 \\
 0 & 1 & 1 & | & -2 & 1 & 0 \\
 0 & 5 & 7 & | & -3 & 0 & 1
 \end{array}
 \begin{array}{l}
 \curvearrowright \\
 \curvearrowright \cdot (-5)
 \end{array}$$

$$\begin{array}{ccc|ccc}
 1 & 0 & -1 & | & -1 & 1 & 0 \\
 0 & 1 & 1 & | & -2 & 1 & 0 \\
 0 & 0 & 2 & | & 7 & -5 & 1 & :2
 \end{array}$$

$$\begin{array}{ccc|ccc}
 1 & 0 & -1 & | & -1 & 1 & 0 \\
 0 & 1 & 1 & | & -2 & 1 & 0 \\
 0 & 0 & 1 & | & 7/2 & -5/2 & 1/2
 \end{array}
 \begin{array}{l}
 \curvearrowright \cdot (-1) \\
 \curvearrowright
 \end{array}$$

$$\begin{array}{ccc|ccc}
 1 & 0 & 0 & | & 5/2 & -3/2 & 1/2 \\
 0 & 1 & 0 & | & -11/2 & 7/2 & -1/2 \\
 0 & 0 & 1 & | & 7/2 & -5/2 & 1/2
 \end{array}$$

$$A^{-1} = \frac{1}{2} \begin{pmatrix} 5 & -3 & 1 \\ -11 & 7 & -1 \\ 7 & -5 & 1 \end{pmatrix}$$