

Integriere mit Hilfe einer Substitution:

$$\int_0^{\frac{\pi}{2}} \sin(2x) dx$$

$$u = 2x \quad \frac{du}{dx} = 2 \Rightarrow dx = \frac{du}{2}$$

$$\text{untere Grenze} = 2 * 0 = 0$$

$$\text{oberer Grenze} = 2 * \frac{\pi}{2} = \pi$$

$$\begin{aligned} \frac{1}{2} \int_0^{\pi} \sin(u) du &= -\frac{1}{2} [\cos(u) \Big|_0^{\pi}] = -\frac{1}{2} [\cos(\pi) - \cos(0)] = -\frac{1}{2} [-1 - 1] \\ &= 1 \end{aligned}$$

$$\int e^{(2x+1)} dx$$

$$u = 2x+1 \quad \frac{du}{dx} = 2 \Rightarrow dx = \frac{du}{2}$$

$$\int \frac{1}{2} e^u du = \frac{1}{2} e^u + c = \frac{1}{2} e^{2x+1} + c$$

$$\int_0^a \cos(x + \pi) dx$$

$$u = x + \pi \quad \frac{du}{dx} = 1 \Rightarrow du = dx$$

$$\text{Untere Grenze} = 0 + \pi = \pi$$

$$\text{Obere Grenze} = a + \pi$$

$$\int_{\pi}^{a+\pi} \cos(u) du = \sin(u) \Big|_{\pi}^{a+\pi} = \sin(a + \pi) - \sin(\pi) = \sin(a + \pi)$$