

Integriere mit einem geeigneten Verfahren:

$$\int_0^{\pi} x \sin(x) dx = -x \cos(x) \Big|_0^{\pi} - \int_0^{\pi} -\cos(x) dx = -(-\pi - 0) + \sin(x) \Big|_0^{\pi} = \pi$$

(partielle Integration)

$$\int_0^5 3ae^{3x} dx \quad (\text{Substitution})$$

$$u=3x \quad \frac{du}{dx} = 3 \Rightarrow dx = \frac{du}{3}$$

$$\text{untere Grenze} = 3 \cdot 0 = 0$$

$$\text{obere Grenze} = 3 \cdot 5 = 15$$

$$\int_0^5 3ae^{3x} dx = \int_0^{15} ae^u du = a(e^u \Big|_0^{15}) = a(e^{15} - e^0) = a(e^{15} - 1)$$

$$\int x \cos(2x) dx \quad (\text{Substitution + partielle Integration})$$

$$u=2x \quad \frac{du}{dx} = 2 \Rightarrow dx = \frac{du}{2}$$

$$\begin{aligned} \int x \cos(2x) dx &= \frac{1}{4} \int u \cos(u) du = \frac{1}{4} \left[u \sin(u) - \int \sin(u) du + c \right] \\ &= \frac{1}{4} [u \sin(u) + \cos(u) + c] = \frac{1}{2} x \sin(2x) + \frac{1}{4} \cos(2x) + c \end{aligned}$$

$$\int \frac{3a}{x} dx = 3a \ln(x) + c \quad (\text{Grundintegral})$$

$$\int_0^1 e^{3a} dx = e^{3a} [x \Big|_0^1] = e^{3a} (1 - 0) = e^{3a} \quad e^{3a} \text{ ist eine Konstante}$$